Mid Semester Examination

Physics III,
B. Math., 3^{rd} year,
September - December 2022.

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November 16^{th}, 2022, Morning Session.
Duration: 3 hours.
Total points: 100

Please give arguments where necessary. If it is unclear from your answer why a particular step is being taken, full credit will not be awarded. Grades will be awarded not only based on what final answer you get, but also on the intermediate steps.

1. **(4+4)+6+6=20**

   (a) A charge $+Q$ is placed above an infinite grounded conducting plane a distance l above. We want to place a charge $-Q$ at mechanical equilibrium (i.e., zero electrostatic force on $-Q$) somewhere between the plane and the charge $+Q$, along and within the line segment joining $+Q$ to the plane.

   i. Does such a point exist in the spatial range described above? Explain your answer.

   ii. Find the location of $-Q$ for mechanical equilibrium to happen.

   (b) Let a volume $\mathbb{V}$ of arbitrary shape, enclosed by a surface $\mathbb{S}$, carry a uniform charge density ρ. The resultant electric field, everywhere, can be written as an integral over the surface alone! Show that the electric field everywhere can be written as

$$\vec{E}(\vec{r}) = \frac{\rho}{4\pi\epsilon_0} \int_{\mathbb{S}} \frac{d\vec{S}\left(\vec{r'}\right)}{|\vec{r} - \vec{r'}|}$$

   (c) Consider the same geometry as the previous problem but now let the charge density be an arbitrary smooth function of the position inside the enclosed **finite** space $\mathbb{V}$. Show that the electric field everywhere can be written as

$$\vec{E}(\vec{r}) = -\frac{1}{4\pi\epsilon_0} \int_{\mathbb{V}} dV' \frac{\nabla_{\vec{r'}}\rho(\vec{r'})}{|\vec{r} - \vec{r'}|}$$

2. **6+(3+6)=15**

   (a) Using the definition of $\vec{D}$ in terms of $\vec{E}$ and and $\vec{P}$, the electrostatic equations of dielectric matter, and the boundary conditions for $\vec{E}$, **derive** the boundary conditions that $\vec{D}$ satisfies for an arbitrary dielectric surface. Do not assume a simple dielectric.

   (b) Imagine a charge density distribution of total charge Q distributed over small volume V_Q. Similarly, an uncharged simple linear dielectric of total volume V_d has a polarization distribution $\vec{P}(\vec{r}); \vec{r} \in V_d$ with a total dipole moment $\vec{p}$. They are placed at a distance r from each other. Assume that $r \gg V_Q^{\frac{1}{3}}, V_d^{\frac{1}{3}}$.

      i. Find the r-dependence of the force that the dielectric feels from the far-away charged object.

      ii. Is this force attractive or repulsive? Explain your answer physically, in terms of the polarization of the dielectric.

3. **(2+8)+(4+6) = 20**

   (a) An infinitely extended solid cylinder of radius R and magnetic permeability μ_1 has a thin wire along its axis. This wire carries a free current I_f. This whole assembly is now embedded in an infinite magnetizable medium of permeability μ_2.

      - Using Ampere's Law, find the magnetic field everywhere.
      - Do the same by explicitly calculating the magnetic fields separately due to the free currents and bound currents and verify that the results are identical.

   (b) A solid sphere of radius R has uniform magnetization $\vec{M}$.

      i. Draw the field lines for $\vec{B}$ and $\vec{H}$ both inside and outside the sphere.

      ii. Show, by direct calculation, that both of them satisfy matching conditions at the spherical surface.

4. **3+6+8+6+2=25**

   The quasi-magnetostatic situation, analogous to the quasi-electrostatic situation discussed in class, is the case when an external current fluctuates **slowly** in time (the physical condition for what **slowly** means in this context is the same as that of quasi-eletrostatics). Mathematically, this amounts to neglecting the displacement current term in the Ampere-Maxwell equation, assuming it to be small compared to the external current. The other three Maxwell's equations remain unchanged.

   Consider an infinitely long solenoid with n turns per unit length and radius a has a quasi-magnetostatic current given by $I(t) = I_0 \cos(\omega t)$.

(a) Show that in a quasi-magnetostatic approximation, we can only have a time-independent charge distribution, which may, of course be spatially varying.

(b) Find the magnetic field everywhere.

(c) We assume, WLOG (as far as time-dependence is concerned), that there is no charge distribution. Thus, the electric field that appears is purely Faraday-like. Use this information and other symmetry arguments (without actually calculating it) to find the direction and variable dependence of the electric field that appears, both inside and outside the solenoid.

(d) Calculate the electric field everywhere and draw a qualitative plot of its magnitude as a function of distance from the solenoidal axis.

(e) Which symmetry of the Maxwell's equations is violated by the electric and magnetic field configurations outside the solenoid because of the quasi-magnetostatic approximation?

5. **5+5+(4+4+2) = 20**

A long solid dielectric cylinder of radius R is permanently polarized so that the polarization is everywhere radially outward, with a magnitude that increases linearly with the radial distance ρ from the cylindrical axis. Now the cylinder is rotated about its axis with an angular velocity ω without causing any change in the polarization. Treat this problem in a limit where $\omega \ll c/R$, i.e., ignore both the Faraday electric field and the displacement current (this is the non-relativistic limit of electrodynamics, which incorporates both quasi-electrostatics and quasi-magnetostatics simultaneously). Find

(a) The electric field everywhere.

(b) The magnetic field everywhere.

(c) The total energy stored in the electric and magnetic fields both
   - before the cylinder starts spinning.
   - after the cylinder starts spinning.
   - Explain the reason for the difference in field energy in the two parts.